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TIME AND SOURCE ENCODING FOR  
MULTIPLEXED COMPRESSED SIGNALS

by  
Carlo Joseph Broglio

Thesis submitted to the Faculty of the Graduate School  
of the University of Maryland in partial fulfillment  
of the requirements for the degree of  
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1969

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## **ABSTRACT**

**Title of Thesis: Time and Source Encoding for Multiplexed Compressed Signals**

**Carlo J. Broglio, Master of Science, 1969**

**Thesis directed by: Associate Professor Dr. Alan B. Marcovitz**

Sensor data transmitted from a spacecraft to the ground contain much redundancy. Efforts to remove this redundancy, called data compression, tend to be in the form of polynomial predictors in which the data are "curve fitted" to the longest straight line within a prescribed error tolerance. Once compressed, a major problem in reconstructing the data is that of identifying the time of occurrence of each data point.

This thesis proposes five methods for identifying the time of occurrence of a data point. Two methods are simulated on a CDC 3200 computer and the other three are calculated by using a probability of occurrence table measured by applying zero-order and linear predictors to spacecraft data taken from the Orbiting Geophysical Observatory B satellite. A noise-free environment and a prescribed error tolerance were assumed in the process of obtaining the probability of occurrence table. These proposed methods are analyzed and compared by the use of information theory and the results are presented in this Thesis.

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iii

TABLE OF CONTENTS

Chapter	Page
I. INTRODUCTION .....	1
II. THEORETICAL BACKGROUND .....	6
Telemetry .....	6
Data Processing Techniques .....	9
Data Compression .....	9
III. THEORY OF OPERATION .....	16
IV. RESULTS AND DISCUSSION .....	27
V. CONCLUSIONS .....	31
APPENDIX A. DERIVATION OF THE COMPRESSION RATIO FOR SCHEME C .....	35
APPENDIX B. DERIVATION OF THE COMPRESSION RATIO FOR SCHEME D .....	47
APPENDIX C. DERIVATION OF THE COMPRESSION RATIO FOR SCHEME E .....	54
BIBLIOGRAPHY .....	64

## LIST OF TABLES

Table		Page
A1	K Values and Compression Algorithms for the OGO-B Matrix . . .	37
A2	RMS Error Table in Quantization Levels . . . . .	38
A3	Raw Compression and Probability of Occurrence for Each Data Word . . . . .	39
A4	Huffman Code for Number of Samples Skipped, Scheme C. . . . .	42
B1	Huffman Code for the Main Frame Word Positions, Scheme D . . .	49
C1	Huffman Code for the Experiment Sensor, Scheme E . . . . .	56
C2	Comparison of Ambiguity in Scheme E . . . . .	63

## LIST OF ILLUSTRATIONS

Figure		Page
1	A time-multiplexed telemetry pattern . . . . .	8
2	Zero-order predictor compressed waveform . . . . .	11
3	Linear predictor compressed waveform . . . . .	12
4	A received time-multiplexed, compressed telemetry pattern . . . . .	14
5	Simulated compression schemes . . . . .	19
6	Time identification codes for schemes A and B . . . . .	21
7	A data compression system . . . . .	22
8	Data processing methods . . . . .	23
9	Data formats for compression schemes C, D, and E . . . . .	25



## CHAPTER I

### INTRODUCTION

With the advent of satellites, the volume of data obtained and processed each year has increased enormously. For example, one Orbiting Ground Observatory (OGO) transmits to earth over 5 billion binary digits per day, thereby overloading the satellite-to-earth data channel. Consequently, it has become necessary to reduce the data bandwidth necessary for transmission.

One method under consideration is data compression--the removal of redundancy from the data. Redundancy occurs in many forms; *natural redundancy* is that redundancy which is intrinsic to the signal being observed. An example of this is the measurement of temperature, voltage, or current. These parameters tend to remain constant over long periods of time.

*Forced redundancy* is inherent in the design of the spacecraft. Examples of this kind of redundancy are oversampling, subcommutation identification, and spacecraft clocks.

*Correlation redundancy* arises because of its space relationship to other samples. An example of this is television picture data. If one considers the picture an  $(n \times m)$  matrix, each interior point is related to the eight samples surrounding it. This type of redundancy is beyond the scope of this paper.

When data compression is used in a telemetry system, the following problems are encountered:

1. How to remove the redundancy,
2. How to control the error introduced by compression,
3. How to identify the time of occurrence of the data received,
4. How to transmit at nonuniform data rates,
5. How to recover the data when errors are made during transmission.

Furthermore, in a multiplexed telemetry system, there is the additional problem of identifying the sensor that produced the received data quantity.

The problem of redundancy removal has been studied on a one-source-one-output basis in the past (References 1 through 10). Some of the solutions considered in the past are polynomial prediction, polynomial interpolation, functional curve fitting, bit plane encoding, and depiction of data in a periodic manner.

The problem of controlling the introduced error must be solved by the user of the telemetry compression system since he is the one who best knows the limitations of the data. The problem of identifying the time of occurrence of the received data sample has also been studied in the past. Some of the solutions have been run-length encoding, ordering the raw telemetry samples, and numbering the raw telemetry samples.

The nonuniform rate of the compressed data stream (known as the buffering problem) has been the subject of many previous studies. It has been considered mainly a design problem in feedback control system theory (References 2, 3, and 4). The recovery of data after transmission is a problem of error detection and correction coding theory.

The problem of identifying sources in a multiplexed data stream has received very little attention in the past. The solutions proposed so far tend to treat this subject as a time identification problem. Furthermore, the problem of whether to compress the data and then multiplex it, or to multiplex and then compress, has not been investigated.

The object of this paper is to study the problems of redundancy removal, time sequence identification, and sensor identification with the main emphasis on the last two problems. The problem of buffering is not considered because it can be treated better in feedback control theory and the problem of transmission errors is not considered because it is handled best within the context of error correction coding; these two theories are beyond the scope of this paper. The previously mentioned solutions do not represent a complete list of the possible methods considered, but are intended as representative in each problem study area.

The problem of redundancy removal is approached using zero-order and first-order polynomial predictors. These methods were chosen because past studies (References 2 through 4) have shown them to be the most promising methods when applied to the sensor data used throughout this study.

The problem of introducing errors by compression was assumed solved by allowing a maximum average error rate of one quantization level. This assumption is meant to be interpreted as a presentation of results at this error level rather than as a solution to the problem of error control.

Five methods of coding time and sensor identification are compared. This paper differs from past work in that it considers a multiplexed data stream,

suggests a method of data identification based on an existing data processing system, and is based on actual data from the OGO-B spacecraft now orbiting the earth.

The following considerations are of major importance: simplicity in spacecraft design, minimum changes in the current ground recovery hardware, and minimum changes in current data processing. These considerations are necessary to reduce the cost of converting to a data compression system so that the advantages of data compression will not be overcome by the cost of data recovery.

It is assumed that:

1. The data are in a noise-free environment,
2. The compressor is to be flown on the spacecraft,
3. Data from many experiments are to be multiplexed before transmission,
4. The experiments are mutually independent.

The assumption of a noise-free environment is perhaps unrealistic; however, the nature of the source is best studied under this condition, and it is hoped that the results of this study can be used to solve the noise problem, perhaps through coding.

Two types of compression algorithms were used—the zero-order predictor (ZOP) and the linear predictor (LP). A detailed description of their operation appears later. These two algorithms were applied to data from the OGO-B spacecraft, and the results were evaluated to determine a combination that would allow high compression with reasonable error rates. After completion of this phase of the study, attention was focussed on time and sensor identification. Two

methods of time identification were simulated on the CDC 3200 computer, and three others were compared theoretically. The methods simulated on the computer encode the number of samples skipped within the frame matrix as identification, whereas the theoretical methods encode each position of the frame matrix in one case and each source in the other. In all cases, the frame synchronization code is assumed to be transmitted uncompressed. These methods will be discussed in detail later.

The frame matrix was maintained throughout this study because of the simplicity in applying orbit data to the frame sequence to determine the exact position of the spacecraft at the time the data were transmitted.

## CHAPTER II

### THEORETICAL BACKGROUND

#### Telemetry

For the purpose of the present research, the telemetry system is a time-sampled digital system with a quantization precision set by a  $k$ -bit binary code. The data source is assumed to be some analog function of time which is sampled at a constant rate.

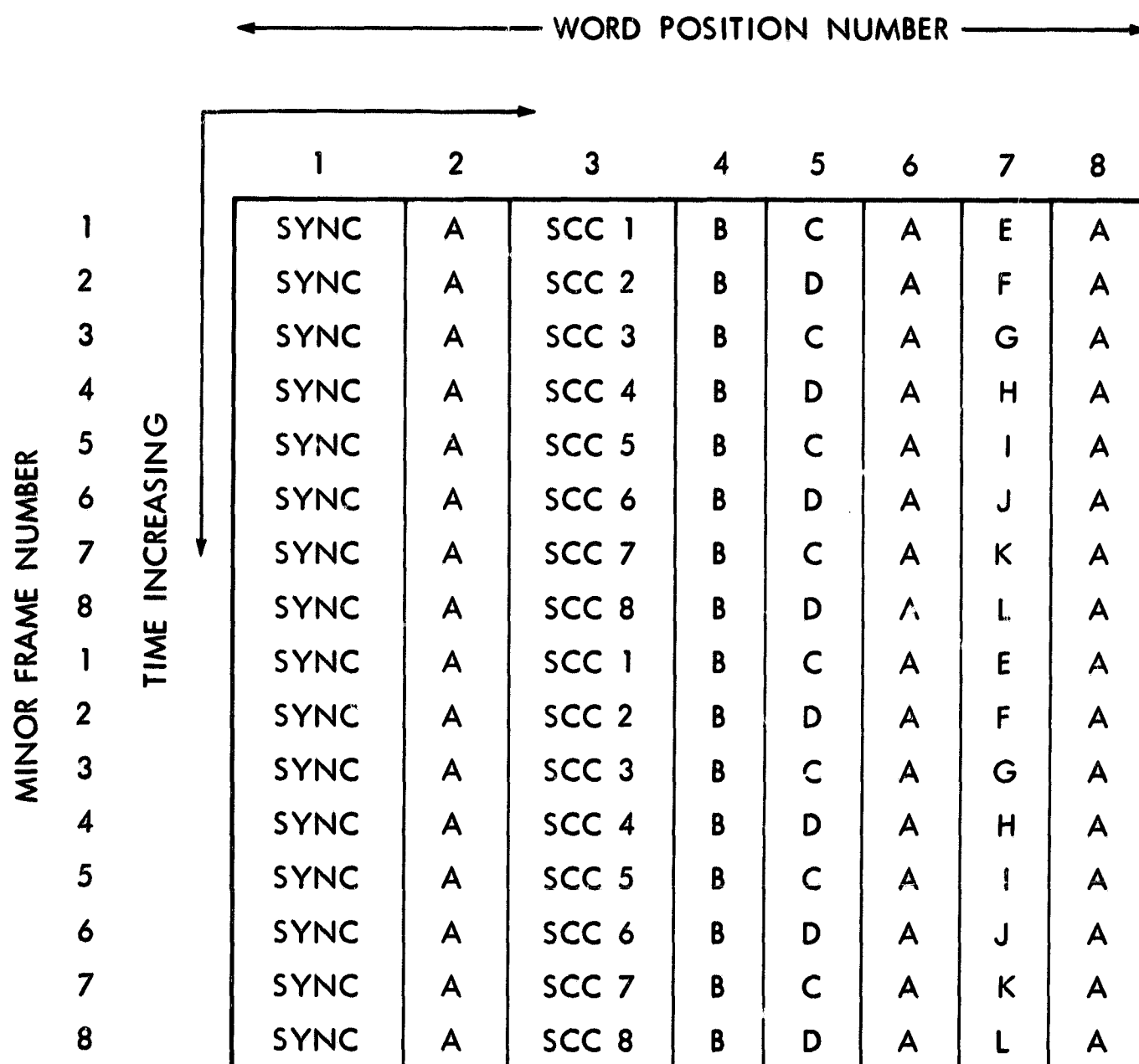
In a time-multiplexed telemetry system, different sources are sampled in a definite order or intermixed, time-shared sampling called *commutation*. The commutator may be mechanical or electronic, and its operation may either be inflexible (according to a predetermined pattern), changeable (according to different patterns, each brought about by ground command) or automatic, depending on the data. The output of a commutator is a sequence of samples from different sources, the pattern of samples repeating in some time period which is typically large in comparison to the time period between samples.

When a time-multiplexed telemetry signal is received, the samples are "sorted" into sources from the time-multiplexed sequence. This process, called *decommutation*, depends on frame synchronization and word synchronization in the multiplexed data sequence for reliable operation. Word synchronization uses an easily recognizable "sync" word that is inserted in the sampling sequence

once every period or fractional period of the multiplexed sequence. The details of a simple, multiplexed pattern with its included word synchronization are illustrated in Figure 1.

In Figure 1, the letters A through L signify data sources such as space experiments, attitude sensors, spacecraft subsystem parameters (voltages, currents, temperatures, etc. associated with the spacecraft and not individual experiments), and an on-board clock. The subcommutator count (SCC) word has its lowest value at the beginning of the longest repetitive cycle in the multiplexed data format and has its highest value at the end of this longest cycle, which is called the "main frame." In Figure 1, the main frame constitutes eight rows of the pattern; two main frames are shown. Each row is called a minor frame, and the synchronization word, called a frame sync pattern (FSP), occurs once each minor frame. Source A, occurring three times each minor frame, is said to be *supercommutated*, that is, sampled more than once each minor frame. Source B is sampled once every minor frame. Sources C and D are sampled once every other minor frame, and these are subcommutated—sampled less than once each minor frame. Sources D through L are also subcommutated, and since their subcommutation pattern has the longest period, this pattern determines the subcommutator count.

Usually each major and minor frame are much larger than in this example. For example, the OGO-B telemetry format (Reference 11) has a  $(128 \times 128)$  major frame matrix, that is, 128 minor frames, each with 128 samples.



A, B, C, D, E, F, G, H, I, J, K, L = SAMPLED SOURCES

SYNC = SYNCHRONIZATION WORD

SCC = SUBCOMMUTATION COUNT

Figure 1 — A time-multiplexed telemetry pattern.



## Data Processing Techniques

After the data are time-multiplexed, they are transmitted to the ground tracking stations and recorded on tape. The first stage of processing is to obtain "bit sync" and "frame sync" from the recorded bit stream. For this purpose, the computer later uses analog-to-digital processors. The output of these processors is called a "buffer tape" that contains the information transmitted to the ground in a computer-acceptable format.

The computer analyzes the FSP to determine the data error quality, thereby providing data quality control. The computer also merges the data with orbit position data calculated from the spacecraft clock and orbit projection programs. These data are then stored on tape in the data archives. This tape, called an *edit tape*, is the input to the next stage of processing, decommutation. In decommutation and subsequent data processing, it is necessary only to locate absolute time as a reference point somewhere within a main frame: the absolute times of all samples can then be derived from their positions in the main frame (see Figure 8a).

## Data Compression

There are several methods for compressing data (Reference 3). Perhaps the simplest is *polynomial prediction*, in which an  $n$ -th order polynomial is generated by the compressor using  $(n + 1)$  consecutive samples. The next sample is derived by evaluating the polynomial. This prediction is then compared with the current data value. If the current data value is within  $\pm K$  of the predicted value, the sample is not transmitted. The value of  $K$  is a parameter of the

compression method and is chosen by the experimenter, based upon the acceptable peak error.

The simplest of these methods is the zero-order predictor (ZOP). The operation of the ZOP is based on predictions of future samples using a horizontal projection of a zero-order polynomial from the present sample (Reference 4). This method simply adds (or subtracts) the K value, which establishes a peak error, to the present sample. As long as subsequent samples fall within this range, they are considered redundant and are not transmitted (Figure 2). The value of the subsequent sample is then assumed by the receiver to be the same as the present sample and to fall on a horizontal line projected through the sample. When a future sample falls outside this range, it is transmitted as a nonredundant sample. The K value is then set around the new sample, and the process is repeated.

The second method to be considered is *linear prediction* (Reference 1). The linear predictor (LP) uses a first-order extrapolation polynomial of the form

$$y_t = y_{t-i} + (y_{t-i} - y_{t-i-1})(i) \pm K,$$

where  $y_{t-i}$  is the last sample sent and  $y_{t-i-1}$  is the value prior to  $y_{t-i}$  assumed by the receiver. Thus, if the previous sample was not transmitted, the predicted value of  $y_{t-i-1}$  is used.

The extrapolation equation is a straight line drawn between the last two data points. Initially, the first two data points are transmitted, and a straight line is drawn through them. An aperture of width  $2K$  is placed about the straight line (Figure 3). If the new data point is within  $\pm K$  of the predicted value, then that point is not transmitted. If the new data point is outside the aperture, then that

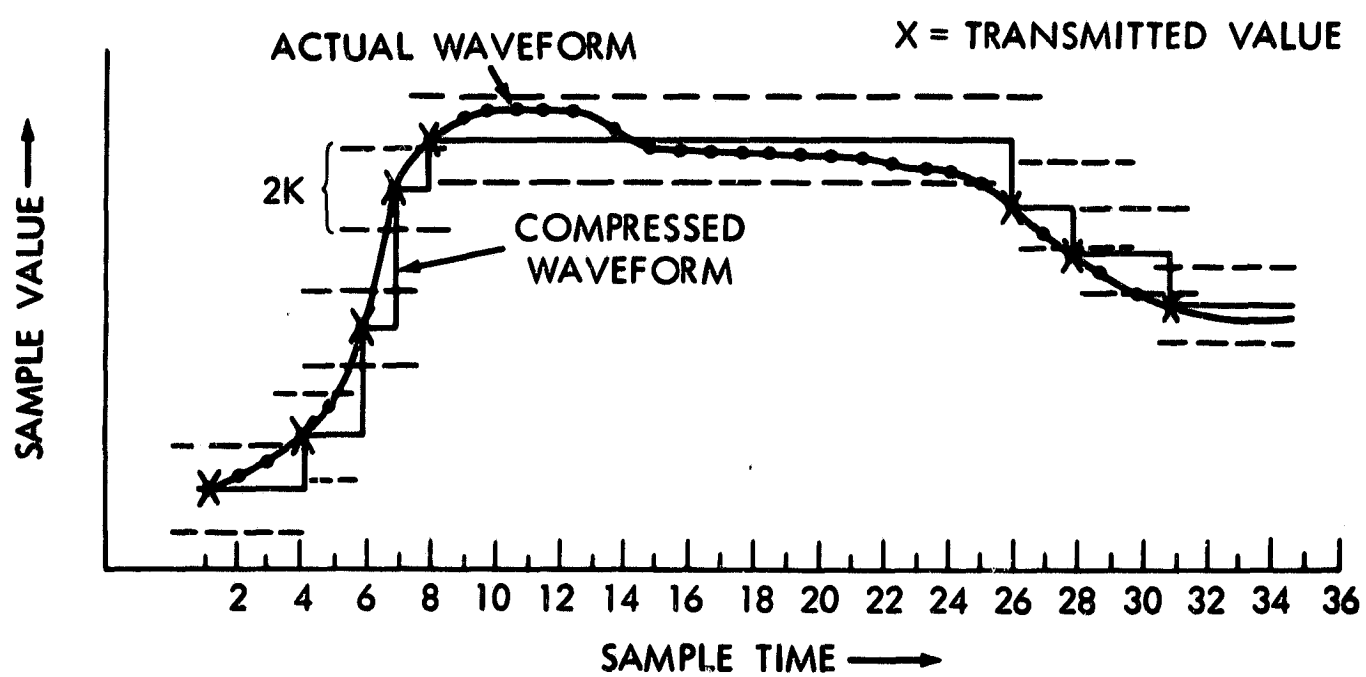


Figure 2 — Zero-order predictor compressed waveform.

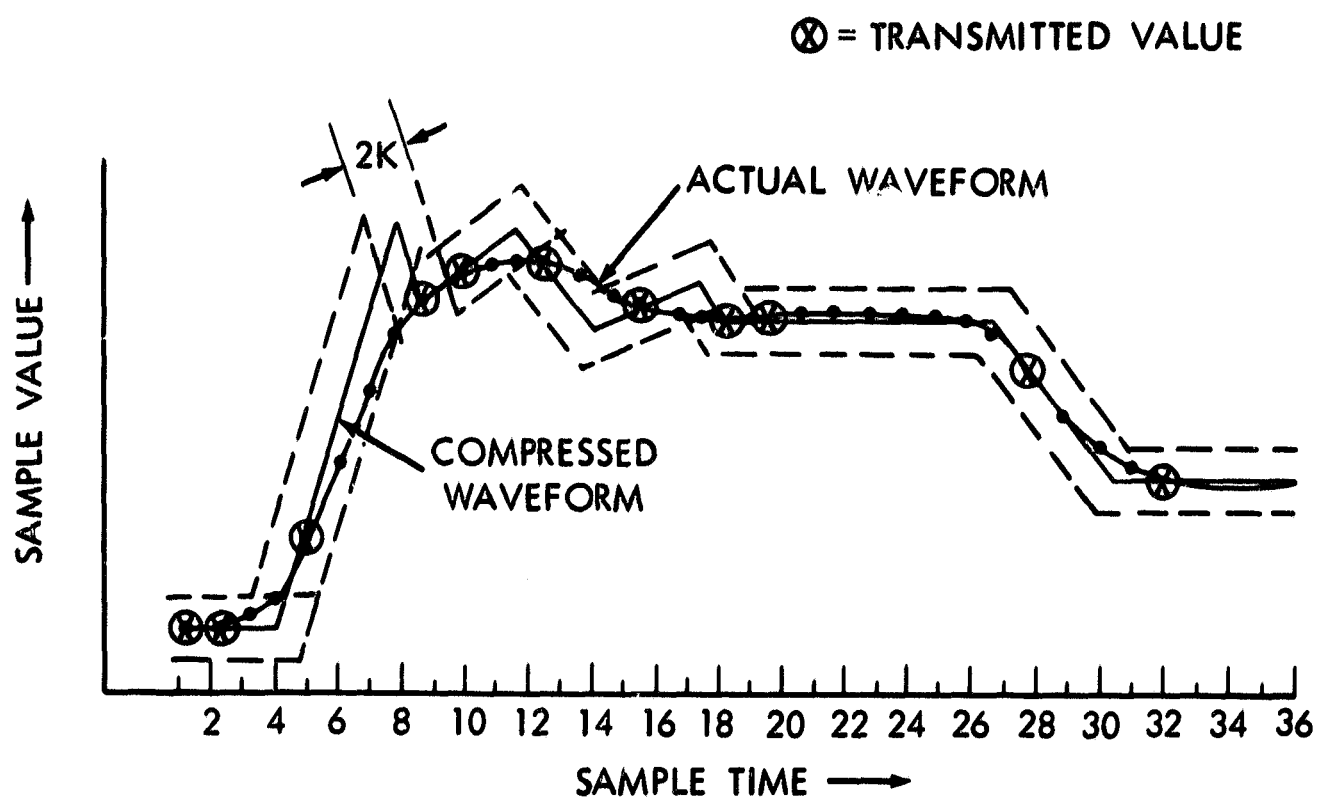


Figure 3 – Linear predictor compressed waveform.

point is transmitted, and a new prediction line is drawn through the present data point that was transmitted and the previously predicted data point.

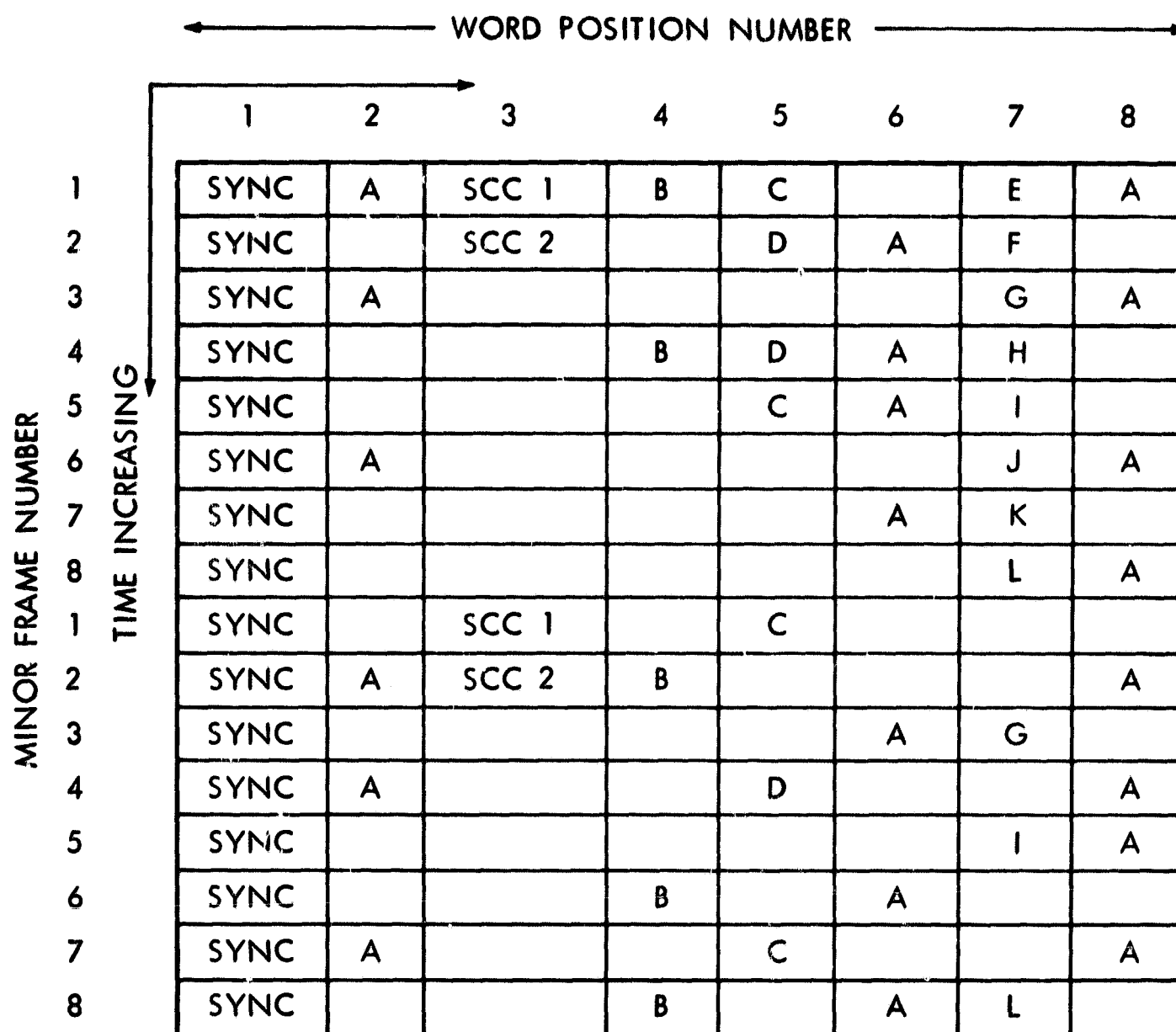
Assigning a tolerance value  $K$  to the compression algorithm subjects the quantized waveform to error in reconstruction. Assume that the value of  $K$  is 3 (Figures 2 and 3); then any sample that falls within three or less counts of the predicted value would not be sent. Thus, an error is introduced into the transmitted data value (Reference 3).

The methods of data compression mentioned previously are by no means a complete list of possibilities (see References 12 through 21). They were chosen because of their ease of implementation and data reconstruction. These methods offer the most promise when the impact on ground data processing is considered.

When data compression is used, the continuity of the input data waveform is lost. Hence, a requirement to reestablish the space relationship of the data is imposed on the compressed data stream. This requirement is satisfied by the use of a time identification encoding scheme. The choice of such a scheme fixes the cost of identifying the data.

When a multiplexed data source is considered, the problem increases because now not only the continuity of the data source must be reconstructed, but also the source itself must be identified. Figure 4 shows the telemetry pattern in Figure 1 after compression. The blank slots represent missing data points caused by compression.

In minor frame 1, some means of identifying the fact that word 6 is missing is necessary. This may be done by identifying the word position number of each data word sent, by identifying the source of each data word sent, or by ordering



A, B, C, D, E, F, G, H, I, J, K, L — SAMPLED SOURCES  
 SYNC — SYNCHRONIZATION WORD  
 SCC — SUBCOMMUTATOR CC SET

Figure 4 — A received time-multiplexed compressed telemetry pattern.

the data words and identifying the number of words skipped since the last transmitted sample.

Thus, each word sent could have a 3-bit prefix representing a binary number of its position in the minor frame (for example, word 4 of minor frame 1 would be 100A), or it could have a 4-bit source identification followed by a 2-bit repetition factor where necessary: That is, each of the 12 data sources A through L could be represented by a unique 4-bit pattern, and for sources appearing more than once (such as A), the number of the output for that source in that frame could appear. For example, word 8, frame 1, could be prefixed by 000111, where 0001 represents source A, and 11 represents the third output of source A in frame 1.

Finally, the number of words since the last transmitted data word could be encoded into a binary number of fixed length such as four bits, and sent with the next transmitted sample. For instance, in the first appearance of minor frame 8, the telemetry word 7 that is transmitted could be prefixed by 0101. These methods and others will be studied in more detail later.

## CHAPTER III

### THEORY OF OPERATION

Two terms necessary for comparing compression algorithms are raw compression ratio and actual compression ratio. The raw compression ratio is the number of data bits transmitted divided by the number of data bits in the uncompressed data stream. The actual compression ratio is the number of data bits plus identification bits transmitted divided by the number of data bits in the uncompressed data stream. The second value includes the cost of identifying the data.

Five compression schemes will be considered. In all five cases, a set of samples  $xi_1, xi_2, xi_3, \dots, xi_n$  in a minor frame is sent; that is,  $n \leq 128$  samples of the frame. They are ordered such that  $1 \leq i_1 \leq i_2 \leq i_3 \leq \dots \leq i_n \leq 128$ .

In the first three schemes A, B, and C, advantage is taken of the ordering; in the last two schemes D and E, it is not. Hence, from information theory, one would expect schemes A, B, and C to be better, since schemes D and E have a larger range to encode than the other three. That is, it is more probable that some of the 128 words will appear rather than that all 128 will be skipped. This hypothesis will be examined later.

First, the two compression schemes that were submitted to computer simulation will be examined. Both schemes employ the idea of minor frames and word



position in the frame in data recovery, as is presently done. Both attempt to identify the time of occurrence in the spacecraft multiplexer rather than at the experiment sensor. Time information is recovered by transmitting every FSP. Also, both methods employ a parameter T which allows T-compressed major frames to elapse, then transmits a full major frame of uncompressed data, which allows all measurements to be reestablished without error. In the case of constants such as voltages and currents, which would be absent for hours, T allows a periodic check on their value. Another use of the parameter T is to increase the redundancy when the noise level of the spacecraft-to-ground communication channel is increased because of electrical storms, solar activity, etc.

In the computer simulation, a 7-bit word was inserted after the FSP to represent the number of words appearing before the next FSP. This required the spacecraft to store an entire frame of data before transmission and minimized the need for ground equipment modification since it enabled the equipment to predict the time of occurrence of the next FSP. This removes some of the burden of data error control from the computer since it can be preprocessed to a limited extent in the frame synchronizer (Reference 22). Since this 7-bit word could be added to any of the schemes to be discussed, it will be omitted from the analysis of the schemes.

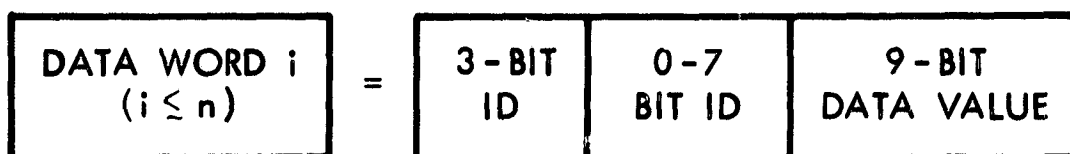
The first scheme (A) is designed for minimal analog-to-digital equipment modification. Each transmitted data word in scheme A comprises a 3-bit identification section, a 0- to 7-bit tag, and a 9-bit data value. The 9-bit value is the level of quantization used by OGO-B (Reference 23).

The 3-bit identifier represents a binary count of the number of bits in the variable-length section following it. This enables the data processor to count the number of bits in a word and thereby separate data words (see Figure 5). This method was chosen because it represents only a slight change in the current data processors that are designed for uncompressed data streams.

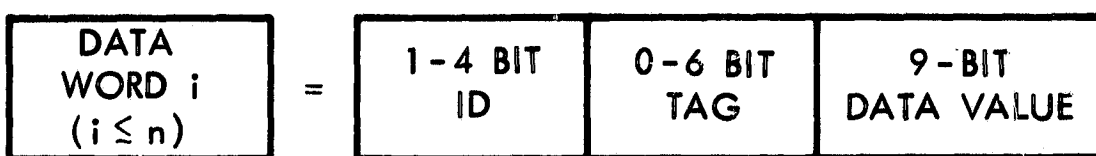
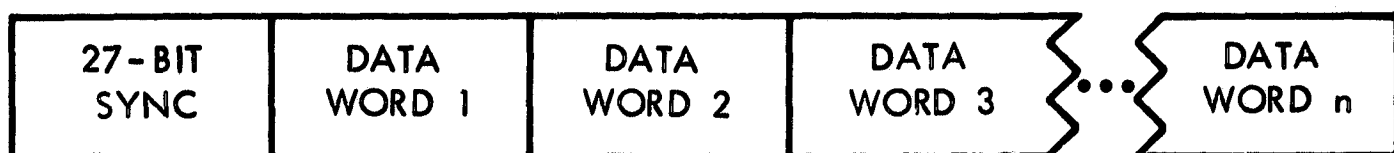
The 0- to 7-bit tag represents the number of data words missing from the minor frame format since the previous data word. This enables the computer to reconstruct the frame format; further processing may then be done with no change in existing programs. Hence, time and source identification are accomplished in the same way as they are now done.

The second scheme (B) is similar to scheme A except that the word identification is modified. The transmitted data word is now composed of a 1- or 4-bit identifier, a 0- to 6-bit tag and a 9-bit data value. The first bit of the identifier represents whether or not any data words have been omitted from the minor frame format. If its value is zero, then no words are absent. If it is a one, however, the next three bits represent the number of bits in the tag section and also the scale factor for the tag section.

The 0- to 6-bit tag section represents a binary value that must be added to 2 raised to the power represented by the last three bits of the identifier to obtain the number of words skipped. For example, if one word is skipped, the data identifier is 1000; for two words, the data identifier is 10010; and, for 125 words, the data identifier is 1110111101. Analyzing the last case reveals: 1 signifies that some word(s) has been skipped; 110 signifies that six bits are in the tag and that the value of the tag must be added to  $2^6$ , or 64. The 6-bit tag 111101 is



(a) Scheme A Compressed Frame



(b) Scheme B Compressed Frame

Figure 5 – Simulated compression schemes.

decoded as  $1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 61$ . Thus,  $61 + 64 = 125$  words are skipped (see Figure 6).

The implementation of these schemes is exemplified in Figures 7 and 8. In Figure 7, the dashed-line enclosure represents the additional amount of design and development needed in the spacecraft. The dashed line encloses a large portion of the control box, signifying the necessity for increased complexity in this area, and the portion outside the dashed line corresponds to the current design.

Figure 8a represents the current methods for recovery of the transmitted data; Figure 8b shows the increased effort necessary to handle compressed data in the current system. The new box represents merely a subprogram added to the current buffer tape processing programs. Neither scheme will require additional equipment at the receiving station. However, both schemes require minor modifications of the current analog-to-digital data processors.

A theoretical calculation of the worst case, that is, all words of a minor frame present, shows the following actual compression ratios:  $C_A = 0.767$  and  $C_B = 0.926$ , where  $C_A$  refers to scheme A and  $C_B$  refers to scheme B. These calculations were based on the OGO-B spacecraft format, which was used throughout the study. These schemes are optimal when the probability of not skipping a word is 0.5 for scheme B and 0.125 for scheme A and for both schemes when the probability of skipping one word is  $1/16$ ; skipping two or three words is  $1/32$ ; skipping four to seven words is  $1/64$ ; skipping eight to 15 words is  $1/128$ ; skipping 16 to 31 words is  $1/256$ ; and skipping 64 to 125 words is  $1/1024$ . Computation of these probabilities showed, however, that the foregoing schemes were not optimal for the data source

Number of words skipped	Scheme A code		Scheme B code		
	ID	Tag	Flag	ID	Tag
0	000	—	0	—	—
1	001	0	1	000	—
2	001	1	1	001	0
3	010	00	1	001	1
34	110	100010	1	101	00010
35	110	100011	1	101	00011
125	111	1111101	1	110	111101
126	111	1111110	1	110	111110

Figure 6 – Time identification codes for schemes A and B.

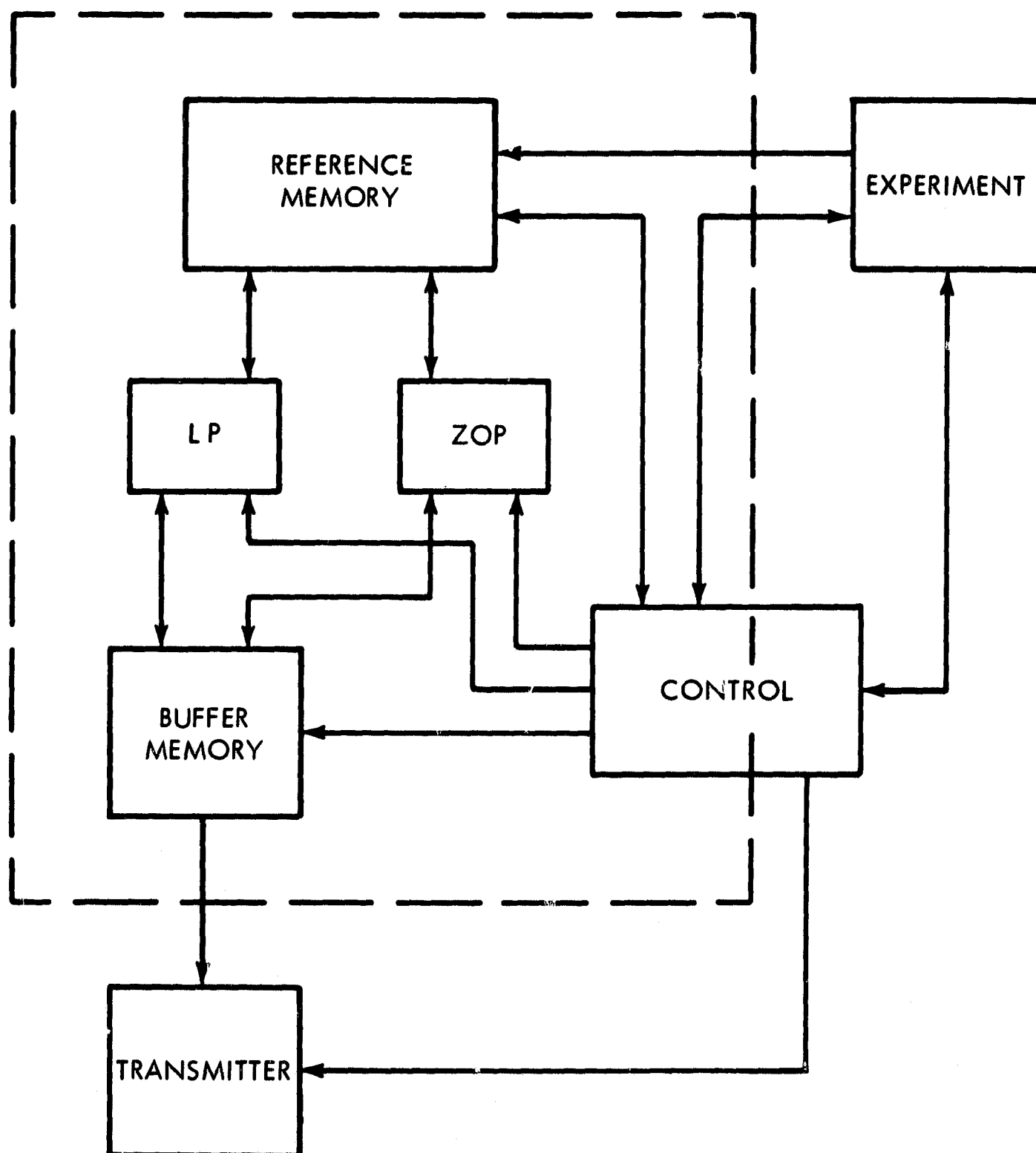
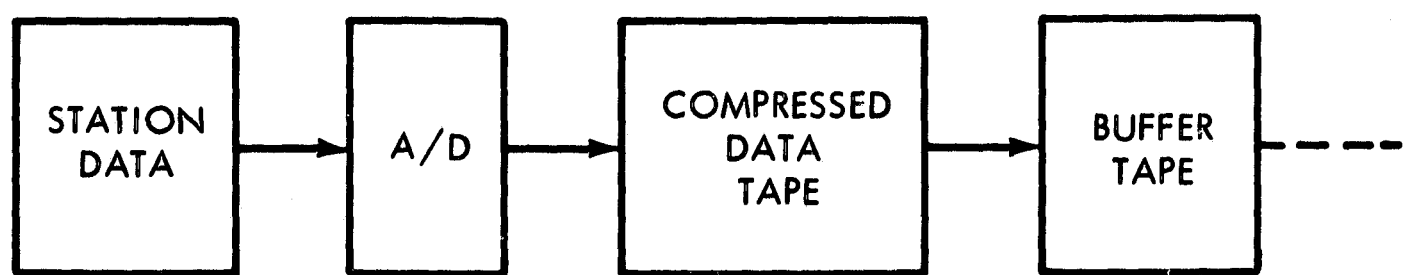
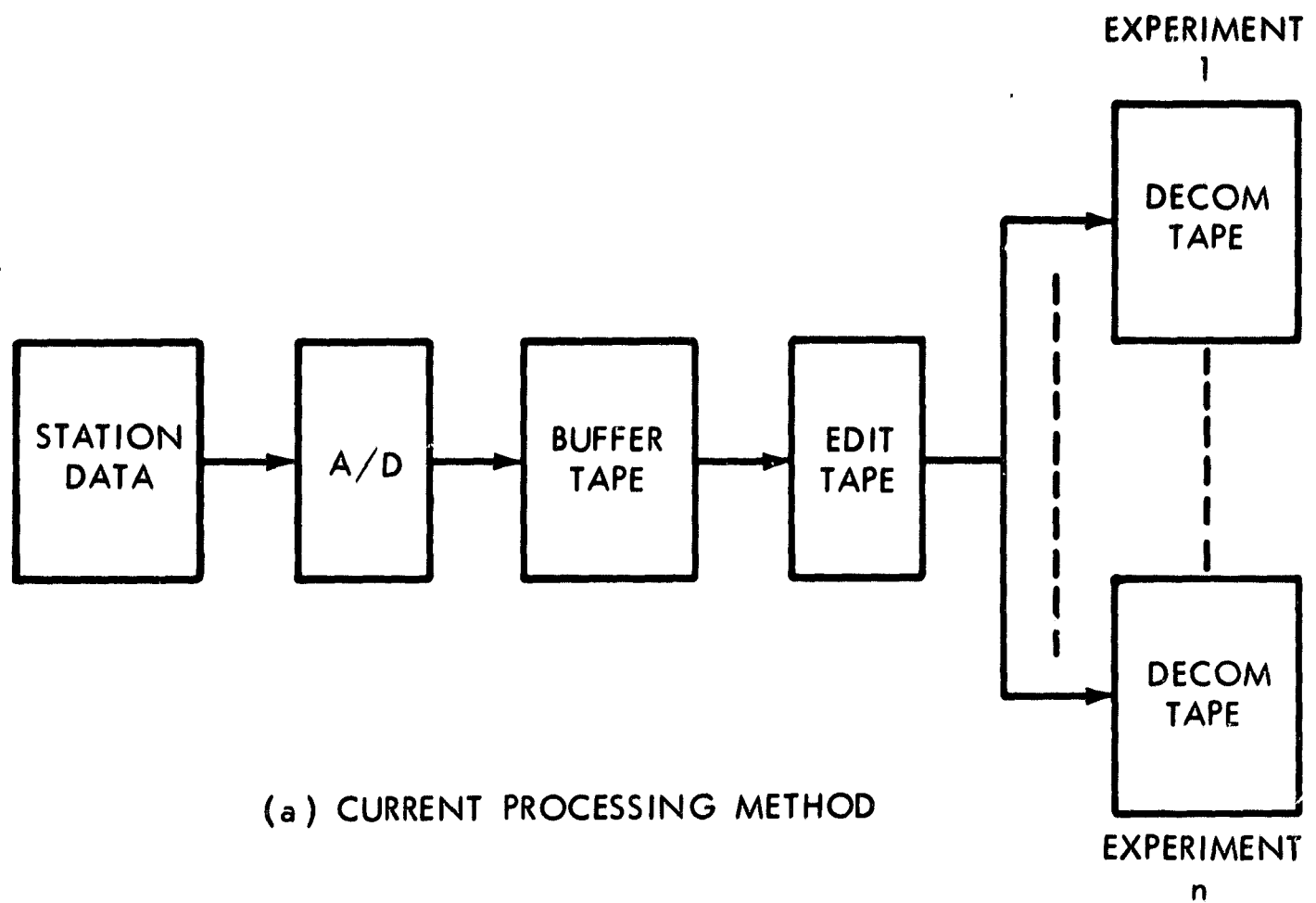


Figure 7 – A data compression system.



(b) COMPRESSION PROCESSING METHOD

Figure 8 – Data processing methods.

used. The results of this computation appear in Table A4. Since the source was neither of these forms, scheme C was derived as an optimal code by using the Huffman method (Reference 24). The probabilities used were derived from the word compression ratios measured by the computer for schemes A and B. Hence, scheme C represents an optimal code for methods A and B. Scheme C was not simulated on the computer; its derivation appears in Appendix A. Figure 9a shows a frame format using scheme C as the time identification algorithm.

The last two theoretical methods, which were not simulated on the computer, as mentioned earlier, will be described next. The first method (D) employs a probability of occurrence table to derive an optimum code for each word of the frame (see Appendix B). This table is derived by observing the raw compression ratio for each word. The data words of the minor frame are assumed to be independent in their occurrence although compression is done on an experimental basis. Hence, if an experiment's output occurs in more than one word, then all words belonging to the experiment are compressed as a single source. Thus, each output from the source is assumed to be independent of the previous output from that source.

The probability of occurrence for a word is found by:  $(1/C_{Ri}) = P_{Ai}$ , where  $C_{Ri}$  is the raw compression ratio for word  $i$  and  $P_{Ai}$  is the probability that word  $i$  occurs in a frame. Since these values represent the word activity, they do not sum to unity. The table may be normalized by dividing each value by

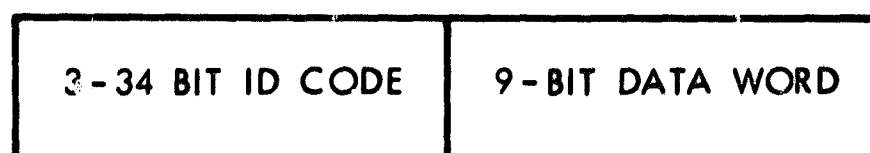
$$\sum_{i=1}^{128} P_{Ai}$$

The derivation of the code from this table is described in Reference 24.

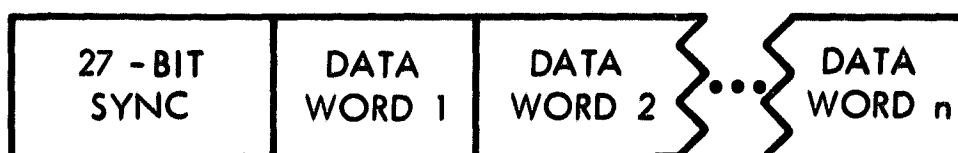




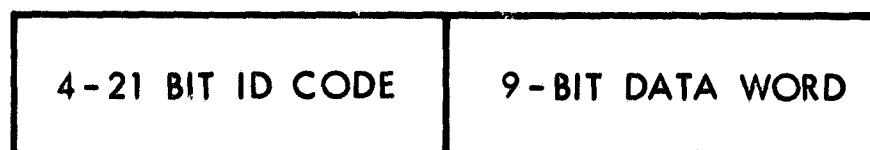
WHERE DATA WORD  $i$  =



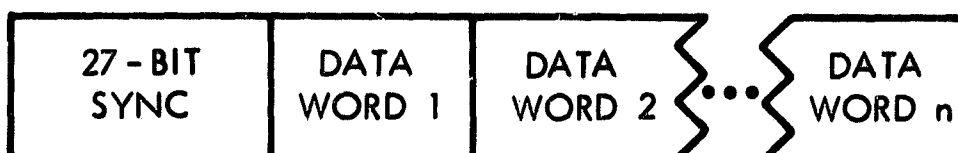
(a) Scheme C



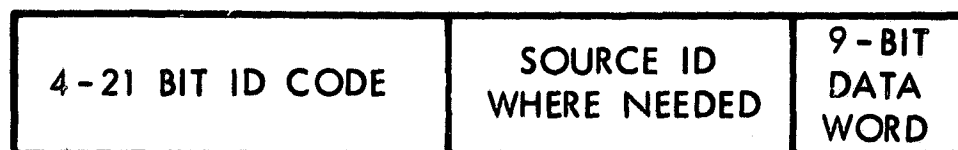
WHERE DATA WORD  $i$  =



(b) Scheme D



WHERE DATA  
WORD  $i$  =



(c) Scheme E

Figure 9 – Data formats for compression schemes C, D, and E.

Now each word has its own label that defines it every time the word appears (see Figure 9b). Once the word is defined, the frame can be reconstructed, and processing is identical to current methods.

The last method (E) attempts to find an optimum code for each experiment sensor, based on a probability of occurrence table for the experiment sensor (see Appendix C). This table is found by summing all the compressed output bits for each sensor and dividing by the total number of bits for that sensor in the uncompressed data stream, yielding a raw compression ratio for each sensor. The probability of occurrence is then found as in scheme C, and the code identification derived. As each word is transmitted, as required by the compression algorithm, this identification tag, modified where necessary, is also sent (see Figure 9c).

For sensors with more than one word per frame, an ambiguity can arise concerning which word is being sent. This problem is increased when the position of the sensor words in the main frame is such that the words are close together. In order to minimize the possibility of confusing sensor outputs, additional bits are added to the foregoing code when the possibility of confusing the outputs is greater than 0.6 percent. Hence the frame can again be reconstructed but at the expense of increasing the reconstruction program complexity.

## CHAPTER IV

### RESULTS AND DISCUSSION

The first phase of the study shows that the ZOP achieves a higher raw compression ratio with less errors than does the LP, for most of the sensors. These values were derived by subjecting the real spacecraft data output to each compression algorithm for K values, ranging from zero to ten over the same time period. The K values were chosen in such a manner as to provide reasonable error rates (less than one quantization level RMS error) while yielding good compression. These K values are shown in Table A1 along with the type of algorithms used.

Table A2 represents the RMS error introduced for each data word in the minor frame. Note that data words 1, 2, 3, 97, 98, and 99 do not appear here. Words 1, 2, and 3 are the FSP, which is not compressed, and words 97, 98, and 99 are the subcommutator data words, which represent many different sensors. Also notice that certain words appear with K values of zero. These words represent the spacecraft status, clock, binary subcommutator counter, and certain experiments for which no transmission errors can be allowed. These data words are predictable or constant most of the time.

Having arbitrarily set an average error level of less than one quantization level, it will be assumed to be a reasonable one. This fixes the peak error rates for the purposes of this study by selecting  $K$  values for each sensor. However, the experimenter must choose these  $K$  values because he has a clearer understanding of the limitations of his instruments and the usefulness of his data.

Now the problem becomes one of choosing an identification scheme for each data point. For the data used, the five previously described schemes show the following overall compression ratios: scheme A = 5.47, scheme B = 5.58, scheme C = 5.97, scheme D = 5.58, and scheme E = 5.55.

The foregoing results for schemes A and B were obtained by computer simulation. In order to compare schemes A and B with the other schemes, a theoretical calculation was accomplished by use of the following equation: the average number of identification bits for a particular word,  $k$ , equals  $(n_i \times P_i) / 4000$ , where  $n_i$  represents the number of bits needed to encode the fact that  $i$  words were skipped prior to word  $k$ , and  $P_i$  is the probability of skipping  $i$  words prior to word  $k$ ;  $P_i$  is found from

$$P_i = P_{Ak} [1 - P_{A(k-1)}] [1 - P_{A(k-2)}] \cdots [1 - P_{A(k-i+1)}] [P_{A(k-i)}]$$

where  $P_{Ak}$  is the probability of occurrence for word  $k$  and is equal to  $1/C_k$ , where  $C_k$  is the raw compression ratio of word  $k$ . The number 4000 represents the number of minor frames over which the sample was taken.

The results of this calculation show  $C_A = 5.44$  and  $C_B = 5.55$ —close agreement with the simulated results. These values are found under the assumption that the probability of occurrence of a particular word is independent of the probability of the word just prior to it. This assumption seemed to be valid since the

computed compression closely agrees with the simulated ones. Since schemes A and B are not optimal, an optimal code was derived for the data source. The data for this derivation and the code appear in Appendix A. Scheme C represents the result of calculating the compression for this code. As expected, this scheme shows an improvement in compression over the nonoptimal methods.

The hypothesis suggested earlier that schemes A, B, and C are better than schemes D and E is not completely valid. Although the optimum method used in scheme C did yield a slight improvement over schemes D and E, the improvement was not significant. On the other hand, schemes A and B, since they were not optimum, yielded a poorer compression ratio than schemes D and E. This is explained by the fact that the probability distribution of the number of words skipped was more evenly distributed than expected.

Since much of the past work on data compression concerned time-identification of a single source, the last two schemes were used to compare the effects of carrying this work over to a multiplexed source. In scheme D, each word of the minor frame is treated as a data source for time-identification purposes. In scheme E, each sensor is identified separately in the multiplexed data stream.

The data for these derivations and the respective codes appear in Appendix B for scheme D and Appendix C for scheme E. These schemes are representative of past study on a single-source model, and they yield about the same compression as schemes A and B. Further, when scheme C is considered, it is seen that schemes D and E are suboptimal.

Also in scheme E, one encounters the problem of ambiguous data since a sensor with more than one output in a minor frame may appear twice with no

other sensor data separating them in the compressed data stream. Hence, when this source is received, one may not know which output the data represent. The result quoted for scheme E represents a probability of ambiguity of 0.006. It is assumed that this is an acceptable level of ambiguity. Table C2 represents the effect on compression of removing individually those sensors that have a possible ambiguous output. Table C2 also shows the different probabilities of ambiguity associated with each ambiguous sensor.

## CHAPTER V

### CONCLUSIONS

The use of premultiplexer compression offers promise because the nature of the source is most prominent at this point. The need for transmitting only the most meaningful information is dramatized by the present and foreseen telemetry limitations at planetary distances. With reasonable ground facilities, an 8-foot antenna reflector would permit less than 50 bits per second to be transmitted to earth from a spacecraft at 5 astronomical units distance. Even with foreseen improvements in the telemetry system, this figure would only increase to a few thousand bits per second. Since such a telemetry system cannot be allowed to overload with redundant data, some form of compression will be essential.

Two types of compression algorithms were studied, the ZOP and the LP. Table A1 shows that the ZOP was heavily favored (only two LP's out of 128). Actually this figure is deceptive since Table A1 refers to the frame matrix, and several sensors appear more than once in this matrix. A more realistic figure is two LP's out of 89 sensors.

Hence, the cost of implementing an LP seems to be too great, considering its usage. It would be better to sacrifice compression for this reduction in cost. This argument is strengthened further by the fact that both sources that use an

LP are binary counters and hence generate a saw-tooth waveform. Thus, they only need to be transmitted at their zero crossing points. It would be less expensive to implement such a transmitting function than an LP compressor.

Notice also in Table A1 that words 36, 101, and 107 have infinite K values. This is true because word 101 is a fill word in the OGO-B frame format used to keep the frame matrix constant; words 36 and 107 were stipulated by the experimenter as useless to him at 64,000 bps transmission, which was the case here. Thus, an advantage of a compression system is to remove useless data from the transmitted data stream.

This may seem a trivial case. Actually it is not since generally not all experiments on a spacecraft of this type are turned on at all times. The reasons for this are basically three-fold. First, because of the design of certain experiments and the need for saving weight in spacecraft design, the drain on the spacecraft power supply would be too great to permit continuous operation. Secondly, because of the nature of certain experiments and the spacecraft orbit, it is impossible to derive useful data throughout the entire orbit.

The third but less important reason is that because of the unknown factors of space parameters, certain experiments, when flown for the first time, are found to be designed below their useful range. An example of this was Dr. Van Allen's first flight package designed to measure energetic particles. Unaware at the time of the presence of the now famous belts, his instruments went to their maximum value and remained.

Five data methods compression have been proposed. In choosing which one to apply, the cost of implementation versus the compression must be kept in mind.



Implementing schemes C, D, and E would require the development of an entirely new system in the analog-to-digital phase of the processing. Also, extensive reprogramming would have to be done to handle the compressed data in these forms. The spacecraft design would be very complex compared to the current design. Therefore, long development times with great initial cost would be expected.

Schemes A and B, on the other hand, take full advantage of the current data processing programs since only a subprogram is needed to get back to the frame matrix. The frame matrix is important at present because of its usage in time-correlation of the experimenter's data relative to the orbit data. Also, it is important because existing edit and decom programs are based on it.

Furthermore, scheme A could be handled on existing equipment without modification, and scheme B requires only minor modifications. Scheme C, which represents an optimum coding scheme for schemes A and B, would require extensive modification. The slight gain in compression that it offers doesn't warrant its usage in a practical system.

Since the initial cost of an untried data compression system is one of the major deterrents to its usage, schemes A and B are proposed as a practical, first step toward more sophisticated methods. Their moderate initial cost would allow studies of compression algorithms to be made on the spacecraft. Then when optimum data compression methods are found, a gradual change to better time encoding schemes could follow. Thus, the development of better ground systems could be spread out over years while the advantages of data compression were being used in a somewhat more limited sense.

The results show that encoding the number of samples skipped is at least equivalent to encoding each source when considering a multiplexed data stream. In the optimal case, this method is slightly better than source identification, and the ease of implementing it compared to identifying each source, particularly for schemes A and B, leads to the conclusion that this type of time identification is better suited to multiplexed data.

Further, it has been shown that schemes A and B are at least equivalent to the more optimum methods; i.e., the improvement in compression of the optimum methods (C, D, E) is not significant. This fact is of major importance when it is considered that the latter three methods are very dependent on the probability distribution of the data, which may not be known in advance.

## APPENDIX A

### DERIVATION OF THE COMPRESSION RATIO FOR SCHEME C

The data of Table A4 were obtained from the first phase of the study, the selection of K values and compression algorithms. The total number of data bits needed to be output over 4000 frames was found to be 554,274. The total number of data bits in 4000 frames is 4,608,000, yielding a raw compression ratio of 8.31.

Table A4 represents the normalized, ordered probability of skipping the number of words in the leftmost column. These probabilities were found from the equation

$$P_k = \frac{\sum_{i=1}^{128} P_{ki}}{\sum_{k=1}^{128} P_k},$$

where  $P_{ki}$  equals the probability of skipping k words prior to word i;  $P_{ki}$  is found from

$$P_{ki} = P_{Ai} [1 - P_{A(i-1)}] [1 - P_{A(i-2)}] \cdots [1 - P_{A(i-k-1)}] P_{Ak},$$

where  $P_{A_k}$  is the probability of occurrence for word K and is equal to  $1/C_k$ , where  $C_k$  is the raw compression ratio of word K.

The identification code is derived by use of the Huffman Algorithm from Noiseless Coding Theory. The code and the associated number of identification bits are in Table A4.

The net cost of identifying the data points is found from the summation

$$\sum_{i=1}^{128} n_i P_{k_i} 4000 .$$

This total is added to the number of data bits, yielding

$$554,274 + 218,094 = 772,368$$

total bits necessary for transmission.

Hence, the compression is computed from

$$C_c = \frac{4,608,000}{772,368} = 5.96 .$$

Table A1

K Values and Compression Algorithms for the OGO-B Frame Matrix

	0	1	2	3	4	5	6	7	8	9
0	-	0	0	0	1Z	1Z	1L	1Z	7Z	2Z
1	7Z	0Z	3Z	1Z	7Z	7Z	10Z	1Z	1Z	1Z
2	1Z	1Z	1Z	1Z	1Z	1Z	7Z	3Z	1Z	1Z
3	2Z	4Z	6Z	7Z	7Z	10Z	$\infty$	7Z	7Z	10Z
4	2Z	3Z	7Z	1Z	2Z	7Z	4Z	7Z	7Z	1Z
5	1Z	1Z	1Z	1Z	2Z	7Z	7Z	7Z	0Z	0Z
6	1Z	7Z	7Z	10Z	7Z	0L	0Z	0Z	3Z	2Z
7	2Z	7Z	2Z	3Z	7Z	3Z	3Z	1Z	3Z	5Z
8	3Z	1Z	3Z	2Z	1Z	7Z	7Z	7Z	10Z	1Z
9	1Z	1Z	2Z	2Z	2Z	4Z	6Z	-	-	-
10	1Z	$\infty$	4Z	7Z	1Z	3Z	7Z	$\infty$	3Z	3Z
11	4Z	7Z	7Z	1Z	1Z	1Z	1Z	7Z	7Z	10Z
12	7Z	1Z	1Z	1Z	1Z	2Z	3Z	2Z	2Z	-

Z = ZOP algorithm

L = LP algorithm

Table A2

RMS Error Table in Quantization Levels

	0	1	2	3	4	5	6	7	8	9
0	-	0	0	0	0.114	0.418	0	0.419	1.056	0
1	1.085	-	0	0.122	1.15	1.22	1.49	-	-	0
2	0.024	0.103	0.352	0.414	0.0272	0.16	1.22	0.424	0.315	0.454
3	0.222	0.67	0.899	0	0	0	-	1.305	1.299	1.457
4	0.479	0	1.22	0.01582	0	1.193	1.06	1.148	1.33	0.164
5	0.0134	0.0126	0.162	0.486	0.364	0.67	0.825	0.345	0	0
6	0	1.29	1.25	1.455	1.113	0	0	0	0.564	0.1395
7	0.5	0.419	0.585	0	1.145	0.54	0	0.126	0.525	0.296
8	0.377	0.0475	0.308	0.2318	0.0725	1.015	1.3	1.285	1.456	0.269
9	0.01582	0.491	0.156	0.0675	0.372	0.688	0.915	-	-	-
10	0.498	-	0.685	1.089	0.0174	0	1.08	-	0	0.528
11	1.185	1.17	1.286	0.01582	0.354	0.418	0.0524	1.39	1.312	1.528
12	1.023	0.465	0.05	0.0475	0.0593	0.431	0.42	0.295	0.0226	-

Table A3

**Raw Compression and Probability of Occurrence  
for Each Data Word**

<b>Data Word</b>	<b>Compression</b>	<b>Probability</b>	<b>Data Word</b>	<b>Compression</b>	<b>Probability</b>
1	1.00	1.000000	25	19.13	0.052270
2	1.00	1.000000	26	7.42	0.134770
3	1.00	1.000000	27	210.52	0.004750
4	2000.00	0.000500	28	5.81	0.172100
5	4000.00	0.000250	29	26.84	0.037257
6	$\infty$	0.000000	30	4000.00	0.000250
7	4000.00	0.000250	31	4000.00	0.000250
8	8.31	0.120300	32	95.23	0.010500
9	8.06	0.124100	33	$\infty$	0.000000
10	9.87	0.101317	34	$\infty$	0.000000
11	$\infty$	0.000000	35	27.39	0.036510
12	60.60	0.016501	36	$\infty$	0.000000
13	16.06	0.062266	37	8.43	0.118620
14	7.44	0.134408	38	1.17	0.854700
15	1.13	0.884955	39	6.15	0.162600
16	6.51	0.153610	40	4000.00	0.000250
17	$\infty$	0.000000	41	9.75	0.102564
18	$\infty$	0.000000	42	9.85	0.101522
19	2000.00	0.000500	43	4000.00	0.000250
20	210.52	0.004750	44	23.25	0.043010
21	4000.00	0.000250	45	6.72	0.148810
22	$\infty$	0.000000	46	7.60	0.131578
23	31.49	0.031756	47	2.34	0.427350
24	4000.00	0.000250	48	1.96	0.510200

Table A3—Continued

Raw Compression and Probability of Occurrence  
for Each Data Word

Data Word	Compression	Probability	Data Word	Compression	Probability
49	17.62	0.056753	73	12.34	0.081037
50	20.00	0.050000	74	9.75	0.102564
51	21.62	0.046253	75	75.47	0.013250
52	800.00	0.001250	76	31.25	0.032000
53	108.10	0.009250	77	16.00	0.062500
54	4000.00	0.000250	78	43.47	0.023004
55	4.11	0.243300	79	30.30	0.033003
56	3.48	0.287360	80	7.72	0.129530
57	17.31	0.057770	81	$\infty$	0.000000
58	86.95	0.011500	82	1000.00	0.001000
59	86.95	0.011500	83	$\infty$	0.000000
60	13.60	0.073529	84	2000.00	0.000500
61	8.6	0.116279	85	6.42	0.155763
62	1.16	0.862068	86	7.27	0.133868
63	11.39	0.087796	87	1.15	0.869565
64	7.18	0.139275	88	7.98	0.125313
65	62.50	0.016000	89	36.36	0.027502
66	$\infty$	0.000000	90	400.00	0.002500
67	$\infty$	0.000000	91	4000.00	0.000250
68	9.66	0.103500	92	4000.00	0.000250
69	$\infty$	0.000000	93	4000.00	0.000250
70	4000.00	0.000250	94	$\infty$	0.000000
71	1333.33	0.000750	95	$\infty$	0.000000
72	800.00	0.001250	96	102.56	0.009750



Table A3—Concluded

Raw Compression and Probability of Occurrence  
for Each Data Word

Data Word	Compression	Probability	Data Word	Compression	Probability
97	4.72	0.211800	113	2000.00	0.000500
98	3.79	0.263800	114	4000.00	0.000250
99	3.99	0.250600	115	19.23	0.052002
100	37.73	0.026504	116	285.71	0.003500
101	$\infty$	0.000000	117	7.44	0.134408
102	57.97	0.017250	118	1.13	0.884955
103	6.96	0.143673	119	6.75	0.148148
104	19.13	0.052273	120	7.85	0.127388
105	10.41	0.096061	121	285.71	0.003500
106	8.84	0.113122	122	285.71	0.003500
107	$\infty$	0.000000	123	285.71	0.003500
108	86.95	0.011500	124	266.66	0.003750
109	14.38	0.069541	125	181.81	0.005500
110	6.67	0.149925	126	800.00	0.001250
111	2.39	0.418410	127	166.66	0.006000
112	2.03	0.492610	128	16.19	0.061766

Table A4

## Huffman Code for Number of Samples Skipped, Schematic C

Number of Samples Skipped	Probability of Skipping	$n_i^*$	Code
1	0.173235868	3	000
6	0.092404574	3	100
12	0.076578412	4	0011
7	0.070025065	4	0101
2	0.067545370	4	0110
10	0.054338394	4	1101
5	0.047242803	4	1111
11	0.043795023	4	1010
23	0.041078904	5	00100
9	0.039457290	5	00101
8	0.035963183	5	01000
3	0.035715062	5	01110
13	0.031583676	5	11000
14	0.025718843	5	11100
4	0.023928569	5	11101
25	0.023762695	5	10110
24	0.015909861	6	011110
22	0.014325995	6	110010
15	0.014261874	6	110011
18	0.011157012	6	101110
19	0.009553423	7	0100100
17	0.008817485	7	0100110
16	0.008670054	7	0100111
20	0.005256475	7	1011111
21	0.004666306	8	01001010
31	0.004240716	8	01111100

\* $n_i$  = number of bits in the code

Table A4--Continued

## Huffman Code for Number of Samples Skipped, Scheme C

Number of Samples Skipped	Probability of Skipping	$n_i^*$	Code
35	0.002893870	8	10111100
30	0.002415339	9	010010110
32	0.002060067	9	011111010
26	0.002014548	9	011111011
29	0.001392703	9	101111010
33	0.001058470	10	0100101110
27	0.001050231	10	0100101111
36	0.000859126	10	0111111000
28	0.000857378	10	0111111001
39	0.000769617	10	0111111101
34	0.000755170	10	0111111110
37	0.000694837	10	1011110110
40	0.000487073	11	01111110101
38	0.000461838	11	01111110110
41	0.000431112	11	01111110111
49	0.000382027	11	01111111110
47	0.000339404	11	01111111111
42	0.000257572	11	10111101110
44	0.000235083	12	011111101000
48	0.000214510	12	011111110000
43	0.000203769	12	011111110001
50	0.000199969	12	011111110010
46	0.000141640	13	1011110111010
45	0.000135413	13	1011110111011
56	0.000122354	13	0111111010010
55	0.000048512	14	01111111001100

\* $n_i$  = number of bits in the code

Table A4—Continued

## Huffman Code for Number of Samples Skipped, Scheme C

Number of Samples Skipped	Probability of Skipping	$n_i^*$	Code
54	0.000037403	15	011111101001100
53	0.000034501	15	011111101001101
51	0.000028684	15	011111101001110
52	0.000026756	15	011111110011110
59	0.000026387	15	011111110011100
57	0.000014995	16	0111111010011110
58	0.000013779	16	0111111100111110
62	0.000011165	16	0111111100111010
63	0.000011157	16	0111111100111011
61	0.000010342	16	0111111100110100
72	0.000009806	16	0111111100110110
64	0.000009205	17	01111110100111110
60	0.000009205	17	01111110100111111
70	0.000006534	17	01111111001111110
65	0.000005995	17	01111111001101010
71	0.000005547	17	01111111001101110
73	0.000002335	18	011111110011010111
66	0.000002048	18	011111110011011110
69	0.000001878	18	011111110011011111
67	0.000001860	19	0111111100111111100
68	0.000001376	19	0111111100110101100
74	0.000001270	19	0111111100110101101
84	0.000000838	20	01111111001111111101
80	0.000000754	20	01111111001111111110
77	0.000000495	21	011111110011111110100
82	0.000000466	21	011111110011111110110

\* $n_i$  = number of bits in the code

Table A4—Continued

## Huffman Code for Number of Samples Skipped, Scheme C

Number of Samples Skipped	Probability of Skipping	$n_i^*$	Code
83	0.000000424	21	01111111001111111000
79	0.000000397	21	01111111001111111110
76	0.000000270	22	0111111100111111101010
78	0.000000186	22	0111111100111111101110
75	0.000000159	22	0111111100111111110010
96	0.000000122	23	01111111001111111010110
81	0.000000111	23	01111111001111111011110
95	0.000000080	23	01111111001111111111100
88	0.000000079	23	01111111001111111111101
90	0.000000070	23	01111111001111111111110
97	0.000000069	23	01111111001111111111111
85	0.000000056	24	011111110011111110111110
103	0.000000053	24	011111110011111110111111
94	0.000000047	24	011111110011111111001100
89	0.000000047	24	011111110011111111001101
91	0.000000043	24	011111110011111111001111
87	0.000000033	25	0111111100111111101011100
86	0.000000032	25	0111111100111111101011101
102	0.000000024	25	0111111100111111110011100
92	0.000000015	26	01111111001111111010111100
93	0.000000013	26	01111111001111111010111110
100	0.000000009	26	01111111001111111100111010
108	0.000000008	27	011111110011111110101111010
109	0.000000006	27	011111110011111110101111110
101	0.000000005	27	011111110011111111001110111
115	0.000000004	28	0111111100111111101011110110

\* $n_i$  = number of bits in the code

Table A4—Concluded

## Huffman Code for Number of Samples Skipped, Scheme C

Number of Samples Skipped	Probability of Skipping	$n_i^*$	Code
107	0.000000003	28	0111111100111111101011111110
98	0.000000003	28	0111111100111111101011111111
104	0.000000003	28	0111111100111111110011101100
99	0.000000002	29	01111111001111111010111001110
106	0.000000002	29	01111111001111111010011101111
105	0.000000001	30	011111110011111111001110110100
114	0.000000001	30	011111110011111111001110110101
110	0.000000001	30	011111110011111111001110110110
112	0.000000000	34	0111111100111111110011101101110000
113	0.000000000	34	0111111100111111110011101101110001
116	0.000000000	34	0111111100111111110011101101110010
117	0.000000000	34	0111111100111111110011101101110011
111	0.000000000	34	0111111100111111110011101101110100
125	0.000000000	34	0111111100111111110011101101110101
119	0.000000000	34	0111111100111111110011101101110110
118	0.000000000	34	0111111100111111110011101101110111
120	0.000000000	34	0111111100111111110011101101111000
124	0.000000000	34	0111111100111111110011101101111001
122	0.000000000	34	0111111100111111110011101101111010
121	0.000000000	34	0111111100111111110011101101111011
123	0.000000000	34	0111111100111111110011101101111100
126	0.000000000	34	0111111100111111110011101101111101
127	0.000000000	34	0111111100111111110011101101111110
128	0.000000000	34	0111111100111111110011101101111111

\* $n_i$  = number of bits in the code

## APPENDIX B

### DERIVATION OF THE COMPRESSION RATIO FOR SCHEME D

From the raw compression ratios of Table A3, the probability of occurrence table is found from the equation

$$P_{Ai} = \frac{1}{C_{Ri}} \text{ for } i = 4, \dots, 128 .$$

This table is arranged in decreasing order, normalized by the factor

$$\frac{1}{\sum_{i=1}^{128} P_{Ai}}$$

and is presented in Table B1.

The Huffman code and associated code lengths are also presented here. The net cost of identifying the data points for this scheme are found by the summation

$$\sum_{i=4}^{128} P_i n_i$$

In Table B2 the first three words do not appear since they represent the FSP, which was not compressed. However, these 108,000 bits are present in the raw

**data bit count. The result of these calculations yields**

$$554,274 + 270,340 = 824,614 .$$

**The compression ratio is**

$$C_D = \frac{4,608,000}{824,614} = 5.59 .$$



Table B1

## Huffman Code for the Main Frame Word Positions, Scheme D

Main Frame Word Number	Probability of Occurrence	$n_i^*$	Code
118	0.071370951	4	1011
15	0.070846610	4	1100
87	0.069999597	4	1101
62	0.068991248	4	1110
38	0.068708910	4	1111
48	0.040979309	5	10000
112	0.039668455	5	10010
47	0.034445206	5	00000
111	0.033739362	5	00001
56	0.023131529	5	01001
98	0.021276167	5	01110
99	0.020207317	6	100011
55	0.019602307	6	100111
97	0.017081434	6	101010
28	0.013874884	6	001001
39	0.013108539	6	001011
85	0.012564030	6	001100
16	0.012382527	6	001101
110	0.012080023	6	001110
45	0.011999355	6	010000
119	0.011938854	6	010001
103	0.011575848	6	010100
64	0.011233009	6	010110
26	0.010870004	6	010111
117	0.010829670	6	011000
14	0.010829670	6	011001

\* $n_i$  - number of bits in the code

Table B1—Continued

Huffman Code for the Main Frame Word Positions, Scheme D

Main Frame Word Number	Probability of Occurrence	$n_i^*$	Code
86	0.010789336	6	011010
46	0.010607833	6	011011
80	0.010446497	6	011110
120	0.010264994	7	1000100
88	0.010103658	7	1000101
9	0.010002823	7	1001101
8	0.009700319	7	1010000
37	0.009559150	7	1010011
61	0.009377647	7	1010010
106	0.009115476	7	1010110
68	0.008349131	7	0001000
74	0.008268463	7	0001010
41	0.008268463	7	0001011
42	0.008187795	7	0001100
10	0.008167628	7	0001110
105	0.007744121	7	0010000
63	0.007078611	7	0010100
73	0.006534102	7	0010101
60	0.005929093	7	0011110
109	0.005606421	7	0101010
77	0.005041764	8	10001000
13	0.005021579	8	10001001
128	0.004981245	8	10100010
57	0.004658573	8	10100011
49	0.004577905	8	10101110
104	0.004214899	8	00010010

\* $n_i$  = number of bits in the code

Table B1—Continued

## Huffman Code for the Main Frame Word Positions, Scheme D

Main Frame Word Number	Probability of Occurrence	$n_i^*$	Code
25	0.004214899	8	00010011
115	0.004194732	8	00011010
50	0.004033397	8	00011011
51	0.003730892	8	00011110
44	0.003468721	8	00100011
29	0.003004880	8	00111111
35	0.002944379	8	01010110
79	0.002662042	8	01111100
76	0.002581374	8	01111110
23	0.002561207	8	01111111
89	0.002218368	9	101011110
100	0.002137700	9	000111110
78	0.001855362	9	001000100
102	0.001391522	9	010101110
12	0.001331021	9	010110110
65	0.001290687	9	011111011
75	0.001068850	10	1010111110
108	0.000927681	10	0010001010
59	0.000927681	10	0010001011
58	0.000927681	10	0001111110
32	0.000847013	10	0011111000
96	0.000786512	10	0011111010
53	0.000746178	10	0101011110
127	0.000484008	11	00011111110
125	0.000443674	11	00011111111
27	0.000383173	11	00111110010

\* $n_i$  = number of bits in the code

Table B1—Continued

## Huffman Code for the Main Frame Word Positions, Scheme D

Main Frame Word Number	Probability of Occurrence	$n_i^*$	Code
20	0.000383173	11	00111110110
124	0.000302505	11	01010111110
123	0.000282338	12	101011111100
122	0.000282338	12	101011111101
121	0.000282338	12	101011111110
116	0.000282338	12	101011111111
90	0.000201670	12	001111100110
126	0.000100835	13	0011111001110
72	0.000100835	13	0011111001111
52	0.000100835	13	0011111011100
82	0.000080668	13	0011111011110
71	0.000060501	14	00111110111010
113	0.000040334	14	00111110111011
84	0.000040334	14	00111110111110
19	0.000040334	14	00111110111111
4	0.000040334	14	01010111111100
114	0.000020167	15	010101111111010
93	0.000020167	15	010101111111011
92	0.000020167	15	010101111111100
91	0.000020167	15	010101111111101
70	0.000020167	15	010101111111110
54	0.000020167	15	010101111111111
43	0.000020167	15	010101111110000
40	0.000020167	15	010101111110001
31	0.000020167	15	010101111110010
30	0.000020167	15	010101111110011

\* $n_i$  = number of bits in the code

Table B1—Concluded

## Huffman Code for the Main Frame Word Positions, Scheme D

Main Frame Word Number	Probability of Occurrence	$n_i^*$	Code
24	0.000020167	15	010101111110100
21	0.000020167	15	010101111110101
7	0.000020167	15	010101111110110
5	0.000020167	16	0101011111101111
95	0.000000000	20	01010111111011100000
101	0.000000000	21	010101111110111000010
107	0.000000000	21	010101111110111000011
94	0.000000000	20	01010111111011100010
83	0.000000000	20	01010111111011100011
81	0.000000000	20	01010111111011100100
69	0.000000000	20	01010111111011100101
67	0.000000000	20	01010111111011100110
66	0.000000000	20	01010111111011100111
36	0.000000000	20	01010111111011101000
34	0.000000000	20	01010111111011101001
33	0.000000000	20	01010111111011101010
22	0.000000000	20	01010111111011101011
18	0.000000000	20	01010111111011101100
17	0.000000000	20	01010111111011101101
11	0.000000000	20	01010111111011101110
6	0.000000000	20	01010111111011101111

\* $n_i$  = number of bits in the code

## APPENDIX C

### DERIVATION OF THE COMPRESSION RATIO FOR SCHEME E

In Table C1, the sensors are numbered by experiment and letter. The letters represent different sensors of the same experiment. In the cases where a sensor appears more than once in the main frame, the word position numbers of its outputs appear to the left. Table C1 is arranged according to the decreasing order of the probability of occurrence. The probability of occurrence is found by dividing the bits out by the bits in; then a Huffman code is derived for each sensor.

For those sensors with more than one output per frame, the probability of an ambiguous output is calculated by finding  $P_{Ak}$ , where  $P_{Ak}$  is as defined in Appendix A and  $k$  is the number of words between outputs of the sensor. The Huffman code is then modified where necessary so that the probability of ambiguity is less than 0.006. These modifiers appear at the end of Table C1.

Table C2 represents the probability of ambiguity of multiple-output sensors and the effect of added coding to remove this ambiguity on the overall compression ratio. For the case of an ambiguity of less than 0.006, the compression ratio is found by the following procedure. The number of identification bits is found by

the sum

$$\sum_{i=4}^{128} n_i P_i 4000 .$$

To this sum is added the number of data bits required. In Table C2, the first three words representing the FSP do not appear; however, they do appear in the sum of the output bits. The results of these computations show

$$C_E = \frac{4,608,000}{554,274 + 275,351} = \frac{4,608,000}{829,625} = 5.55 .$$

The other compression ratios in Table C2 were computed similarly.

Table C1

Huffman Code for the Experiment Sensor, Scheme E

Main Frame Word Numbers	Sensor Number	Probability of Occurrence	Bits in	Bits out	$n_i^*$	Code
15; 38; 62; 87; 118	10B	0.867550000	180000	156159	3	101**
48; 112	17C	0.499861111	72000	35991	3	011
47; 111	17B	0.422625000	72000	30429	4	1100
56	4B	0.286750000	36000	10323	4	0010
98	SCC2	0.263750000	36000	9495	4	0001
99	SCC3	0.250500000	36000	9018	4	0101
55	4A	0.243000000	36000	8748	5	10000
97	SCC1	0.211750000	36000	7623	5	10011
8; 26; 45; 64; 85; 103; 120	18	0.183500000	252000	34902	5	11110***
28	7B	0.172000000	36000	6192	5	11111
46; 110	17A	0.1406025	72000	10125	5	00000
16; 39; 63; 80; 88; 119	10C	0.134416666	216000	29034	5	00001
14; 37; 61; 86; 117	10A	0.127400000	180000	22932	5	01001
106	3D	0.113000000	36000	4068	6	100100

\* $n_i$  = number of bits in the code



Table C1—Continued

Huffman Code for the Experiment Sensor, Scheme E

Main Frame Word Numbers	Sensor Number	Probability of Occurrence	Bits in	Bits out	$n_i^*$	Code
68	15E	0.103500000	36000	3726	6	110101
74	3C	0.102500000	36000	3690	6	110110
42	3B	0.101500000	36000	3654	6	111000
10	3A	0.101250000	36000	3645	6	111001
9; 73; 41; 105	13A	0.100875000	144000	14526	6	111010
60	6C	0.073500000	36000	2646	6	001101
109	15J	0.069500000	36000	2502	6	001111
77	5B	0.062500000	36000	2250	6	010000
13	5A	0.062250000	36000	2241	7	1000100
128	12D	0.061750000	36000	2223	7	1000101
57	4C	0.057750000	36000	2079	7	1000110
49	7C	0.056750000	36000	2043	7	1001010
104	5C	0.052250000	36000	1881	7	1101000
25	5E	0.052250000	36000	1881	7	1101001
115	8B	0.052000000	36000	1872	7	1101110
50	7D	0.050000000	36000	1800	7	1101111

\* $n_i$  = number of bits in the code

Table C1—Continued

Huffman Code for the Experiment Sensor, Scheme E

Main Frame Word Numbers	Sensor Number	Probability of Occurrence	Bits in	Bits out	$n_i^*$	Code
51	7E	0.046250000	36000	1665	7	1110110
29	7I	0.037250000	36000	1341	7	0011000
35	SPC3	0.036500000	36000	1314	7	0011100
79	19B	0.033000000	36000	1188	7	0100010
23	2G	0.031750000	36000	1143	8	10001110
89	5F	0.027500000	36000	990	8	10001111
100	9I	0.026500000	36000	954	8	10001110
12; 44; 76; 108	13B	0.025750000	144000	3708	8	11101110
78	19A	0.023000000	36000	828	8	11101111
102	11G	0.017250000	36000	621	8	00111010
65	SCCU	0.016000000	36000	575	8	01000110
75	5D	0.013250000	36000	477	9	100011110
59	6B	0.011500000	36000	414	9	001100100
58	6A	0.011500000	36000	414	9	001100101
32; 96	11C	0.010125000	72000	729	9	001100110
53	7J	0.009250000	36000	333	9	001110110

\* $n_i$  = number of bits in the code

Table C1—Continued  
Huffman Code for the Experiment Sensor, Scheme E

Main Frame Word Numbers	Sensor Number	Probability of Occurrence	Bits in	Bits out	$n_i^*$	Code
127	12C	0.006000000	36000	216	10	00110011110
125	12A	0.005500000	36000	198	10	00110011111
27	15B	0.004750000	36000	171	10	00111011110
20	7H	0.004750000	36000	171	10	0100011100
124	9E	0.003750000	36000	135	10	0100011110
123	9D	0.003500000	36000	126	10	1000111100
122	9C	0.003500000	36000	126	10	1000111101
121	9B	0.003500000	36000	126	10	1000111110
116	9A	0.003500000	36000	126	10	1000111111
90	9F	0.002500000	36000	90	11	00111011110
126	12B	0.001250000	36000	45	12	001110111110
72	15I	0.001250000	36000	45	12	001110111111
52	7F	0.001250000	36000	45	12	010001110100
82	1B	0.001000000	36000	36	12	010001110101
71	15H	0.000750000	36000	27	12	010001111100
113	9G	0.000500000	36000	18	13	0100011111010

\* $n_i$  = number of bits in the code

Table C1—Continued

Huffman Code for the Experiment Sensor, Scheme E

Main Frame Word Numbers	Sensor Number	Probability of Occurrence	Bits in	Bits out	$n_i^*$	Code
84	1D	0.000500000	36000	18	13	01000111111011
19	7A	0.000500000	36000	18	13	01000111101100
4	2A	0.000500000	36000	18	13	01000111101101
114	8A	0.000250000	36000	9	14	010001111011100
93	11F	0.000250000	36000	9	14	010001111011101
92	11E	0.000250000	36000	9	14	010001111011110
91	11D	0.000250000	36000	9	14	010001111011111
70	15G	0.000250000	36000	9	14	010001111111000
54	15D	0.000250000	36000	9	14	010001111111001
43	9H	0.000250000	36000	9	14	010001111111010
40	15C	0.000250000	36000	9	14	010001111111011
24	15A	0.000250000	36000	9	14	010001111111100
21	2E	0.000250000	36000	9	14	010001111111101
7	2D	0.000250000	36000	9	14	010001111111110
5	2B	0.000250000	36000	9	15	0100011111111110
30; 94	11A	0.000125000	72000	9	16	01000111111111110

\* $n_i$  = number of bits in the code

Table C1—Continued

Huffman Code for the Experiment Sensor, Scheme E

Main Frame Word Numbers	Sensor Number	Probability of Occurrence	Bits in	Bits out	$n_i^*$	Code
31; 95	11B	0.000125000	72000	9	17	010001111111111110
11; 17; 18	20	0.000000000	108000	0	20	01000111111111111000****
83	1C	0.000000000	36000	0	20	01000111111111111001
81	1A	0.000000000	36000	0	20	01000111111111111010
69	15F	0.000000000	36000	0	20	01000111111111111011
67	EG2	0.000000000	36000	0	20	01000111111111111111
66	EG1	0.000000000	36000	0	20	01000111111111111110
34	SPC2	0.000000000	36000	0	21	010001111111111111000
33	SPC1	0.000000000	36000	0	21	010001111111111111001
22	2F	0.000000000	36000	0	21	010001111111111111010
6	2G	0.000000000	36000	0	21	010001111111111111011

\* $n_i$  = number of bits in the code

Table C1—Concluded

Huffman Code for the Experiment Sensor, Scheme E

	Sensor Number	Word Position	Modifier	Modified Code	$n'_i$
**	10B	15	000	101000	6
		38	001	101001	6
		62	010	101010	6
		87	011	101011	6
		118	100	101100	6
***	18	8	000	11110000	8
		26	001	11110001	8
		45	010	11110010	8
		64	011	11110011	8
		85	100	11110100	8
		103	101	11110101	8
		120	110	11110110	8
****	20	11	00	0100011111111111100000	22
		17	01	01000111111111111100001	22
		18	10	010001111111111111100010	22

where  $n'_i$  = number of bits in the modified code

Table C2

## Comparison of Ambiguity in Scheme E

Sensor Number	Probability of Ambiguity Removed	Number of Identification Bits Needed	Total Number of Bits Output	Actual Compression Ratio
-	None	221,964	776,238	5.936
10B	0.333	264,017	818,291	5.631
18	0.0115	233,298	787,572	5.850
10C	0.006	231,684	785,958	5.862
10A	0.0011	229,608	783,882	5.878
13A	0.00006	225,192	779,466	5.911
17C	0.00005	225,963	780,237	5.906
17B	0.00003	225,345	779,619	5.911
17A	0.000002	223,089	777,363	5.927
13B	0.000008	222,788	777,062	5.931
-	All	300,236	854,510	5.392

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